Detecting and analyzing higher dimensions via the EM radiation field.

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Electromagnetic radiation decays with $1/r$ in three dimensional space, while the non radiating Coulomb field decays faster with $1/r^2$. The general expressions for any dimension are $1/r^{(d-1)/2}$ for the Radiation and $1/r^{(d-1)}$ for the Coulomb field respectively, where d is the number of spatial dimensions. This means that there is a *dimensional dependent* ratio between the two, and one should expect, due to the $1/r^n$ nature, to be able to measure imprints of any propagation through higher dimensional structures at arbitrary scale down to Planck's scale. We present the rules for radiation resulting from the motion of charged objects at any dimension, checked by extensive numerical simulations. These rules are quite different from the 3d case and provide a toolset to analyze higher dimensional structures. We further present a very useful operator to transform any arbitrary propagator in an x-dimensional space into the corresponding propagator in any y-dimensional space.

I. INTRODUCTION

We will first present the exact and *full* expressions of the photon propagators in space-time of any dimension: The Fourier transforms of $1/q^2$. The use of a numerical lattice simulator has been of invaluable help here: It was used to study the higher order derivatives of the Dirac function which occur in the higher dimensional photon propagators, to cross-check analytically derived normalization constants, and, last but not least, to provide the figures in this document. The simulations can be restricted to a 1+1 dimensional lattice with the use of the radial form of the d'Alembertian:

$$\left(\partial_t^2 - \partial_r^2 - \frac{(d-1)}{r}\partial_r \right)\phi \ = \ 0 \qquad (1)$$

Where d is the number of spatial dimensions. Static solutions of the d'Alembertian, with $\partial_t^2\phi = 0$, are the electro static potentials and fields:

Normalized Electro Static Potential:

$$\mathrm{V}_d(r) \ = \ -\ \frac{1}{d-2}\ \frac{\Gamma(d/2)}{2\pi^{d/2}}\ \left(\frac{1}{r^{d-2}}\right) \qquad (2)$$

Normalized Electro Static Field:

$$\mathrm{E}_d(r) \ = \ \frac{\Gamma(d/2)}{2\pi^{d/2}}\ \left(\ \frac{x_1}{r^d}\ ,\ \frac{x_2}{r^d},\ .....\ ,\ \frac{x_d}{r^d}\ \right) \qquad (3)$$

Where the gamma function $\Gamma(n) = (n+1)!$ The electro static potentials $V_d(r)$ do satisfy the radial d'Alembertian at each and every point except for the point $r = 0$ where there is a singularity which we associate with the electric charge. (or the current in the case of the vector potentials) The components of the electro static fields also satisfy (1) except for $r = 0$ where we now find dipole Dirac functions: The spatial derivatives of the Dirac function representing the charge.

The fields are normalized by dividing them with the surface S_d of the d-dimensional hypersphere, so that the integral of $\epsilon_o E$ through the surface represents the electric charge ρ enclosed by it. This is just *Gauss's Law* extended to higher dimensions. The potentials take up another factor of $1/(d-2)$ as a result of the integration, except for the two dimensional case where the potential becomes a log function. We have used the same scheme to normalize the propagators since the potentials can be derived from the propagators by applying them to a constant charge at rest.

Space-time Photon propagators in higher dimensions.

dimension	space-time propagators
1+1d	$\mathrm{P}_1(t,r) \ = \ \frac{1}{2}\ \mathcal{H}(t^2-r^2)$
1+2d	$\mathrm{P}_2(t,r) \ = \ \frac{1}{2\pi^{1/2}}\ \frac{1}{(-\frac{1}{2})!}\ \left(\frac{1}{t^2-r^2}\right)^{\frac{1}{2}}$
1+3d	$\mathrm{P}_3(t,r) \ = \ \frac{1}{2\pi}\ \delta(t^2-r^2)$
1+4d	$\mathrm{P}_4(t,r) \ = \ \frac{1}{2\pi^{1\,1/2}}\ \frac{1}{(-1\frac{1}{2})!}\ \left(\frac{1}{t^2-r^2}\right)^{1\frac{1}{2}}$
1+5d	$\mathrm{P}_5(t,r) \ = \ \frac{1}{2\pi^2}\ \frac{\partial}{\partial(t^2-r^2)}\ \delta(t^2-r^2)$
1+6d	$\mathrm{P}_6(t,r) \ = \ \frac{1}{2\pi^{2\,1/2}}\ \frac{1}{(-2\frac{1}{2})!}\ \left(\frac{1}{t^2-r^2}\right)^{2\frac{1}{2}}$
1+7d	$\mathrm{P}_7(t,r) \ = \ \frac{1}{2\pi^3}\ \frac{\partial^2}{\partial(t^2-r^2)^2}\ \delta(t^2-r^2)$
1+8d	$\mathrm{P}_8(t,r) \ = \ \frac{1}{2\pi^{3\,1/2}}\ \frac{1}{(-3\frac{1}{2})!}\ \left(\frac{1}{t^2-r^2}\right)^{3\frac{1}{2}}$
1+9d	$\mathrm{P}_9(t,r) \ = \ \frac{1}{2\pi^4}\ \frac{\partial^3}{\partial(t^2-r^2)^3}\ \delta(t^2-r^2)$
1+10d	$\mathrm{P}_{10}(t,r) = \ \frac{1}{2\pi^{4\,1/2}}\ \frac{1}{(-4\frac{1}{2})!}\ \left(\frac{1}{t^2-r^2}\right)^{4\frac{1}{2}}$

II. THE PROPAGATORS

The propagators are the vacuum's response on the disturbance by a (charged) Dirac-pulse at t=r=0.

We have found, both analytically and numerically, that the general expression for the photon propagator in a d-dimensional vacuum is given by:

General Photon Propagator in Space-Time:

$$\mathrm{P}_d(t,r) \;=\; \frac{1}{2\pi^a}\,\frac{\partial^a \mathcal{H}(s^2)}{\partial (s^2)^a} \qquad (4)$$

Where a is $(d-1)/2$, the function $\mathcal{H}$ is the Heaviside step function and s^2 is $t^2 - r^2$. All the propagators are derivatives of the Heaviside step function, either whole, or half (semi-) derivatives. The Heaviside step function represents the one dimensional photon propagator.

The higher dimensional photon propagators are obtained via the differential operator. We will demonstrate further on that this operator is generally applicable to derive higher dimensional propagators from any arbitrary 1d propagator. For instance the Klein Gordon propagator for massive particles in space-time, where J_0 is the Bessel function of the first kind of zero order:

$$\mathrm{P}_d^{KG}(t,r) \;=\; \frac{1}{2\pi^a}\,\frac{\partial^a}{\partial (s^2)^a}\left\{\,\mathcal{H}(s^2) J_o(ms)\,\right\} \qquad (5)$$

If we look at fig 1, then we see that some of the propagators are zero everywhere except on the light cone ($d = 3, 5, 7..$) These are the result of whole derivatives). Others are non zero inside the light cone, ($d = 2, 4, 6..$) coming from the fractional derivatives. We can find the fractional derivatives with the help of the formula for fractional integration.

$$D^{-\nu}\, f(t) \;=\; \frac{1}{(\nu-1)!}\int_0^t (t-\xi)^{\nu-1}\, f(\xi)\, d\xi \qquad (6)$$

Where ν is the (fractional) order by which we want to integrate or differentiate. We start from the first derivative of $\mathcal{H}$, which is the dirac function $\delta(\xi)$. This eliminates the integral since the only value of ξ where the function is non-zero is $\xi = 0$.

$$\mathrm{P}_d(t,r) \;=\; \frac{1}{2\pi^a}\,\frac{1}{(-a)!}\int_0^{s^2} \frac{\delta(\xi)}{(s^2-\xi)^a}\, d\xi \qquad (7)$$

$$\text{gives:}\quad \mathrm{P}_d(t,r) \;=\; \frac{1}{2\pi^a}\,\frac{1}{(-a)!}\,\frac{1}{(t^2-r^2)^a} \qquad (8)$$

This gives us the correct description of the behavior inside the light-cone. The functions satisfy the d'Alembertian in (1), and they produce the right value for d when inserted in (1) with a given a. However, one must carefully interpret the division by zero ($t^2 = r^2$) to retrieve the important Dirac function derivatives on the light-cone.

It is the factor $(-a)!$ which separates the propagators into several different groups. It becomes infinite for whole numbers of a, which is the case with $d = 3, 5, 7, 9....$ These propagators become zero everywhere inside the light-cone: They propagate only on the light-cone itself. The factor $(-a)!$ is positive in half of the fractional cases of a corresponding to $d = 2, 6, 10, 14...$, while it becomes negative for the other half ($d = 4, 8, 12, 16...$). It is finite and positive for the exceptional case of $a = 0$ corresponding to the one dimensional propagator.

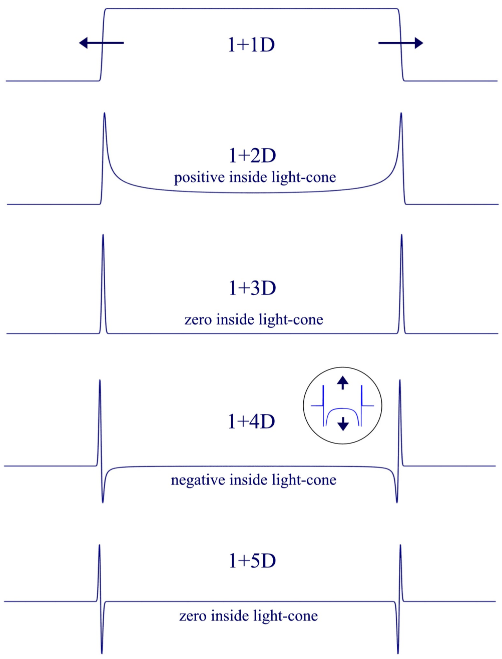

FIG. 1: Simulated photon propagators

III. THE PROPAGATORS AS OPERATORS

The Dirac source used in the simulator has the form $\delta_r(r)\delta_t(t)$. This allows us to define specific (finite) profiles for both the radial and time components. The deltas used are sharp, typically Gaussian, pulses to obtain the propagator. Here however, we use the time dependent profile to represent any time dependent source function. Such a source function would be just a constant for a particle with charge ρ but it can be arbitrary shaped in the case of the vector potential depending on the motion of the particle. The source function can be specifically shaped in order to visualize and analyze the effects of the higher derivatives on such a profile.

fig. 2 shows the response on a $\delta_t(t)$ profile which consist out of three parabolic segments. It appears perfectly smooth but has discontinuities in the third order derivative. As expected, in three dimensions we see the profile itself propagating. In the nine dimensional case however we see the discontinuities revealed as Dirac pulses.

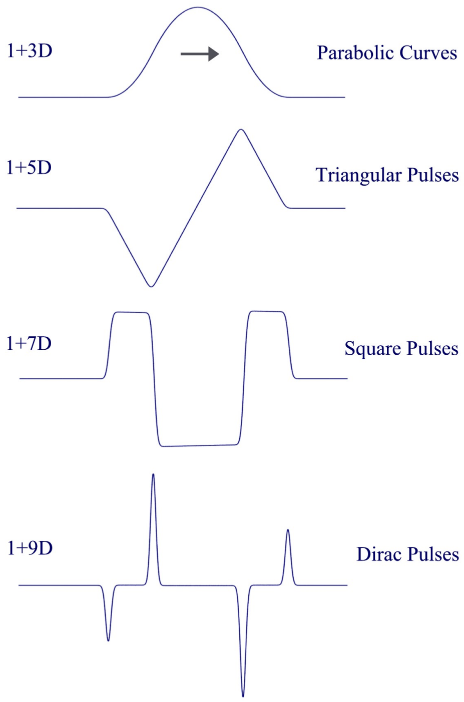

FIG. 2: Simulated propagator response on a source profile constructed with parabolic segments. (see the 3d response)

The 5d propagator is the derivative of a Dirac functions which acts as a differential operator, producing a triangular shaped response. The 7d and 9d propagators are 2nd and 3rd order derivatives of the Dirac function. The 9d response shows Gaussian shaped Dirac pulses. This shape and its width come from the $\delta_r(r)$ profile used as source function. The same Gaussian curve is also visible at the edges of the rectangles.

The simulation shots of fig. 2 are taken sufficiently far from the center to make sure that what we see is mostly from the slowest decaying term. We will see that there are multiple terms from which the Coulomb field term is the fastest decaying term. If the source input is constant, corresponding to a particle at rest, then we *will* see a static Coulomb field at any dimension, even though the propagators represent differential operators. The reason that a constant input is not 100% canceled by the differential propagators is because the propagators, having s^2 as argument, decrease with increasing t. They therefor deviate from the exact derivatives in t.

IV. DERIVATION OF THE PROPAGATORS

We derive the photon propagators by starting with the 1d propagator which is particular simple to derive. The general result is then obtained with the help of our inter dimensional operator:

$$\mathrm{P}_d(t,r) \;=\; \frac{1}{\pi^a}\,\frac{\partial^a}{\partial(s^2)^a}\,\mathrm{P}_1(t,r), \qquad \left(a=\tfrac{d-1}{2}\right) \tag{9}$$

We will prove this operator in the next section where we also will show why and how it works. Now for the derivation of the one dimensional propagator we use coordinate systems which are 45^o rotated: $(t-r, t+r)$ and $(E-p, E+p)$.

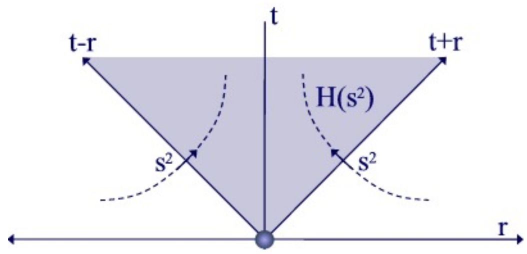

FIG. 3: 1+1d propagator

$$\begin{aligned} \frac{-1}{E^2-p^2} \;&=\; \frac{i}{E-p}\times\frac{i}{E+p} \;\;\Rightarrow \\ \frac{\delta(t-r)\mathcal{H}(t+r)}{\sqrt{2}} \;&*\; \frac{\delta(t+r)\mathcal{H}(t-r)}{\sqrt{2}} \\ &=\; \frac{\mathcal{H}(t^2-r^2)}{2} \end{aligned} \tag{10}$$

The product in momentum space becomes a convolution in configuration space via the *convolution theorem.* The two factors in configuration space are both half-lines, one on the line *t=r* and the other on the line *t=-r.* Both half-lines start on *t=r=0.* The process can also be envisioned as follows:

We start in configuration space with a Dirac pulse at *t=r=0.* We integrate over the *t-r* line which gives us a half-line starting at *t=r=0.* This is the Fourier transform of the first factor in momentum space. A second integration over the *t+r* line gives a 45^o rotated quadrant which gives us our propagator in configuration space, the Fourier transform of the propagator in momentum space.

The argument used for the propagator (s^2), is the correct one with the remark that there is no propagation for $t<0$ as is clearly shown by the derivation. To be complete we could write:

$$\mathrm{P}_1(t,r) \;=\; \frac{1}{2}\mathcal{H}(t)\mathcal{H}(s^2) \tag{11}$$

Generally we will however take the causal propagation at positive time only as understood. The general expression for any dimension takes derivatives along the s^2 lines. These lines, together with the lines where s^2 is constant form an orthogonal coordinate system u,v with:

$$u \;=\; t^2 - r^2, \qquad v \;=\; 2tr \tag{12}$$

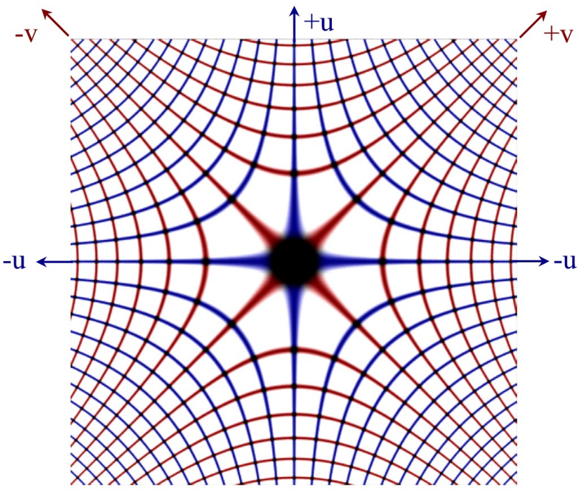

FIG. 4: s^2 coordinate system: $u = t^2 - r^2$ and $v = 2tr$

We may consider this coordinate system as the "square" of the Cartesian coordinate system in the sense of:

$$u + iv \;=\; (t + ir)^2 \tag{13}$$

We are interested in the propagators behavior in terms of t and r rather than in s^2. Specifically in how they work as operators on a time dependent function ϕ_t. The derivatives in s^2 are taken over lines orthogonal to the light-cone. The propagator in 3d is a Dirac pulse $\delta(s^2)$. The "volume" of this pulse decreases with increasing r because the derivative of the argument s^2 increases with time: $\partial_r s^2 = -2r$. The argument goes faster through zero, the Dirac pulse becomes thinner and its volume decreases. We can write for the 3d propagator:

$$\mathrm{P}_3(t,r) \;=\; \frac{1}{2\pi}\delta(s^2) \;=\; \frac{1}{4\pi r}\delta(t - |r|) \tag{14}$$

Where the volume of the Dirac function $\delta(t-|r|)$ is independent of t and r. It produces the standard 3d potential when operating on a constant charge ρ/ϵ_o.

The five dimensional propagator is given by the first derivative of the Dirac pulse: $\partial_{s^2}\delta(s^2)$, which is effectively two Dirac pulses with opposite magnitude. The effect we did see for increasing s^2, when the propagator works as an operator, occurs twice here: The pulses become thinner and they sample less of the source they operate on. The pulses also get spaced less apart: the difference they sample becomes less. The result is that the propagator decays with the square of the distance.

Another effect is that the 5d propagator gets two terms. The two opposite Dirac pulses do not have exactly the same absolute value as a result of the decay. They deviate from the ideal first order differentiator. This deviation can be expressed by a single Dirac pulse and the magnitude is given by the first order approximation of the $1/r^2$ decay: Its derivative. This term is responsible for the 5d Coulomb field which decays with $1/r^3$.

Decay rate of the slowest decaying term:

$$\begin{array}{lllllll}
\text{1d:} & \mathcal{H}(s^2) * \phi_t & = & & \mathcal{H}(t-|r|) * \phi_t & \\
\text{3d:} & \delta(s^2) * \phi_t & = & \frac{1}{t+|r|} & \delta(t-|r|) * \phi_t & \\
\text{5d:} & \frac{\partial\delta(s^2)}{\partial s^2} * \phi_t & = & \frac{1}{(t+|r|)^2} & \frac{\partial\delta(t-|r|)}{\partial(t-|r|)} * \phi_t & + \mathcal{O} \\
\text{7d:} & \frac{\partial^2\delta(s^2)}{\partial(s^2)^2} * \phi_t & = & \frac{1}{(t+|r|)^3} & \frac{\partial^2\delta(t-|r|)}{\partial(t-|r|)^2} * \phi_t & + \mathcal{O} \\
\text{9d:} & \frac{\partial^3\delta(s^2)}{\partial(s^2)^3} * \phi_t & = & \frac{1}{(t+|r|)^4} & \frac{\partial^3\delta(t-|r|)}{\partial(t-|r|)^3} * \phi_t & + \mathcal{O}
\end{array} \tag{15}$$

Higher dimensions get increasingly more terms. The 9d propagator has four terms for instance. If we express these terms in t and r then we find the general expression:

$$\mathrm{V}_d \;=\; \frac{1}{2\pi^a}\sum_{n=0}^{a-1}\left(\frac{\Gamma(a+n)}{\Gamma(n+1)\Gamma(a-n)}\,\frac{1}{(2r)^{a+n}}\,\frac{\partial^b\phi_t}{\partial t^b}\right) \tag{16}$$

(With $b = d - 2 - n$). This expression has been numerically verified by simulations in up to thirteen spatial dimensions. If we write it down explicitly for the 1+9d case we get:

$$\begin{aligned}
\mathrm{V}_9 \;=\; \mathrm{P}_9 * \phi_t \;=\; & \frac{1}{2\pi^4}\,\frac{3!}{0!3!}\,\frac{1}{2^4}\,\frac{1}{r^4}\,\frac{\partial^3\phi_t}{\partial t^3} \\
& + \frac{1}{2\pi^4}\,\frac{4!}{1!2!}\,\frac{1}{2^5}\,\frac{1}{r^5}\,\frac{\partial^2\phi_t}{\partial t^2} \\
& + \frac{1}{2\pi^4}\,\frac{5!}{2!1!}\,\frac{1}{2^6}\,\frac{1}{r^6}\,\frac{\partial\phi_t}{\partial t} \\
& + \frac{1}{2\pi^4}\,\frac{6!}{3!0!}\,\frac{1}{2^7}\,\frac{1}{r^7}\,\phi_t
\end{aligned} \tag{17}$$

The last term corresponds to the Coulomb field. If we replace ϕ_t with a constant ρ/ϵ_o representing a charge ρ at rest, then we obtain for the static Coulomb field by differentiating with respect to r:

Coulomb field in 1+9d space:

$$\mathrm{E}_9 = -\frac{\partial}{\partial r}\mathrm{V}_9 = -\frac{\partial}{\partial r}\left(\mathrm{P}_9 * \frac{\rho}{\epsilon_o}\right) = \frac{105}{32\pi^4}\,\frac{\rho}{\epsilon_o r^8} \qquad (18)$$

Where the normalization factor is one over the surface of a 9-dimensional hypersphere. From here we can write down the general form of the normalization constant and in doing so we recover the surface of an hyper sphere in any dimension:

$$\frac{1}{2\pi^a}\,\frac{\Gamma(d-1)}{\Gamma(a)}\,\frac{1}{2^{d-1}} \;=\; \frac{\Gamma(d/2)}{2\pi^{d/2}} \;=\; \Big(\mathrm{S}_d\Big)^{-1} \qquad (19)$$

V. THE INTER DIMENSIONAL OPERATOR

We will prove here our inter-dimensional operator which derives any propagator in d-dimensional space from the corresponding propagator in 1+1 dimensional space:

$$\mathrm{P}_d(t,r) \;=\; \frac{1}{\pi^a}\,\frac{\partial^a}{\partial(s^2)^a}\,\mathrm{P}_1(t,r), \qquad (a = \tfrac{d-1}{2}) \qquad (20)$$

More generally it can be used to transform an arbitrary propagator in an x-dimensional space into the corresponding propagator in any y-dimensional space.

First we want to look some more at the interrelation of the propagators in different dimensions. We can in general derive, in a d-dimensional space, all the propagators of lower dimensionality by using objects which extend from minus infinity to plus infinity for all the dimensions we want to collapse. The propagator in a d-dimensional space must therefor produce the right 3d propagator or any other lower-d propagator: The propagators of all dimensions are interrelated. We see the same reflected in the Fourier transformation:

The Fourier transform to momentum space is a, spherically symmetrical, convolution with plane waves. Rather than performing a series of 1d fourier transforms for each dimension to obtain the d-dimensional propagator, we can for instance also integrate first over the constant plane of the plane wave, collapsing all spatial dimensions into one, and then do a 1+1d Fourier transform to obtain a result in E and p_1. In this case we derive the 1d propagator.

We can derive the n dimensional propagator by first integrating over (d-n) dimensions of the (d-1) dimensional plane of the plane wave, followed by an 1+n-dimensional Fourier transform.

As stated above: "*We can derive, in a d-dimensional space, all the propagators of lower dimensionality by using objects which extend from minus infinity to plus infinity for all the dimensions we want to collapse*"

Here we find the origin of these derivatives in s^2 or, going from higher to lower dimensions: the integral over s^2 for each step down.

The propagator describes the result of a Dirac type of perturbation of the *entire extended object.* The distance between two points in the lower dimension is only the shortest distance to the extended object which is infinite in size. After the first contribution has come in from the closest point, other contributions from points further away on the extended object will continue to come in indefinitely.

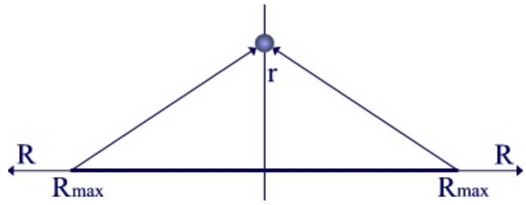

FIG. 5: collapsing the R dimension

This is for instance the reason why the 1d and 2d propagators can be *non-zero* inside the light-cone while in 3d there is no propagation except on the light-cone itself. We need to integrate all the contributions from the extended object, to obtain the lower dimensional propagator. If we collapse a single dimension like in fig.5 then we get:

$$\mathrm{P}_d(t,r) \;=\; 2\int_0^{R_{max}} \mathrm{P}_{d+1}\left(t, \sqrt{r^2+R^2}\right) dR \qquad (21)$$

If we want to collapse multiple dimensions at once than we have to integrate the contributions from an increasingly larger hyper-sphere surface. We get for the general formula:

$$\mathrm{P}_d(t,r) \;=\; \frac{2\pi^{n/2}}{\Gamma(n/2)}\int_0^{R_{max}} R^{n-1}\mathrm{P}_{d+n}\left(t, \sqrt{r^2+R^2}\right) dR \qquad (22)$$

$$r = \sqrt{x_1^2 + x_2^2 ... + x_d^2} \qquad (23)$$

$$R = \sqrt{x_{d+1}^2 + x_{d+2}^2 .... + x_{d+n}^2} \qquad (24)$$

We now proceed with this formula and change the integrating variable to R^2 instead of R. Next we limit the propagation speed to smaller or equal to the lightspeed so we get an explicit expression for the maximum distance R_{max}. We obtain:

$$\mathrm{P}_d(t,r) = \frac{\pi^\nu}{(\nu-1)!}\int_0^{s^2} (R^2)^{\nu-1}\mathrm{P}_{d+n}\left(t,\sqrt{R^2+r^2}\right) d(R^2) \tag{25}$$

with the following substitutions used:

$$\begin{gathered} d(R^2) = 2R\ dR \\ R^2_{max} = t^2 - r^2 = s^2 \\ \nu = n/2 \end{gathered} \tag{26}$$

We can already recognize the shape here of the formula for fractional integration (29). Using s^2 and S^2 as explicit arguments for the propagators we can write:

$$\mathrm{P}_d(s^2) = \frac{\pi^\nu}{(\nu-1)!}\int_0^{s^2} (s^2-S^2)^{\nu-1}\mathrm{P}_{d+n}\left(S^2\right) d(s^2-S^2) \tag{27}$$

with the propagator argument substitutions used:

$$\begin{gathered} P_d(t,r) = P_d(t^2-r^2) = P_d(s^2) \\ P_{d+n}(t,\sqrt{r^2+R^2}) = P_{d+n}(t^2-r^2-R^2) = P_{d+n}(S^2) \end{gathered} \tag{28}$$

This formula is, except for a factor π^ν, equal to the formula for fractional integration we have used earlier, where ν denotes the factor by which we want to integrate or differentiate:

$$\frac{\partial^{-\nu}}{\partial t^{-\nu}}\ f(t) \quad = \quad \frac{1}{(\nu-1)!}\int_0^t (t-\xi)^{\nu-1}\ f(\xi)\ d\xi \tag{29}$$

We used the substitutions $s^2 \to t$ and $S^2 \to \xi$ here. The replacement $d(t-\xi) \to d\xi$ changes the sign twice since it also causes the integration boundaries to be exchanged and thus does not change the net result.

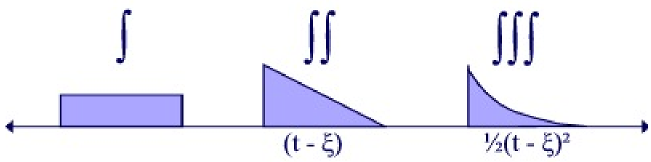

FIG. 6: ν-th order integration via convolution

A ν_{th} order integration is simply a convolution with a $(\nu-1)_{th}$ order kernel, see fig.6. If we replace $-\nu$ with 'a' then we finally retrieve our inter-dimensional operator (20).

VI. THE VECTOR POTENTIALS

We can use our simulator for dynamic effects as long as the perturbation of the position of the charge away from r=0 is small enough. This is indeed the preferable procedure since large displacements will introduce much more complicated effects which only obscure the basic laws we want to visualize.

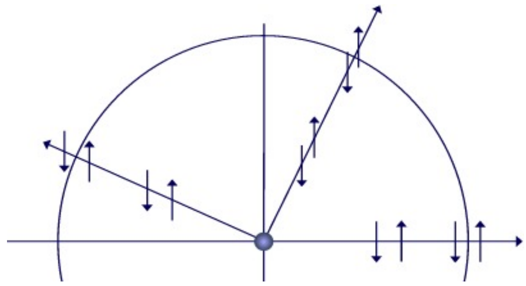

FIG. 7: Vector potential of a vertically perturbed charge.

The higher order derivatives also determine the relation between the magnetic vector potential and the motion of the point charge. Concentrating on the slowest decaying term we have:

Magnetic vector potential of a point charge.

$$\vec{A} \;=\; \frac{1}{2(2\pi)^a\ r^a}\ \frac{\partial^a \vec{x}}{\partial t^a} \;+\; \mathcal{O} \quad \text{where} \quad a = \tfrac{d-1}{2} \tag{30}$$

The behavior of the magnetic vector potential of a point charge in higher dimensions is simply an extension of what we already know from 1d, 2d vector-potentials derived in 3d using classical electrodynamics, for example:

The vector potential, in 3d, on a fixed distance of an infinite plane carrying a constant current keeps rising linear indefinitely according to classical electrodynamics, simply because contributions from points on the plane further away keep coming in indefinitely. The contributions stay constant over time since they come from an ever increasing circle on the plane and the 3d propagator decays with 1/r.

If the current is a Dirac function $\delta(t)$, corresponding to an instantaneous displacement $\bar{x}_d$ of a charged infinite plane at $t = 0$, then the vector potential is given by a Heaviside step-function where the height of the step is proportional to the displacement $\bar{x}_d$. This Heaviside step-function is nothing but the 1d photon propagator

For the response of higher-d propagators we have a look at fig. 2 again. In this case the responses represent the vector potential resulting from a charge which is displaced over a small distance dy at t=0. The velocity during the displacement is defined by parabolic curves.

The 3d vector potential is proportional to the velocity, the 5d vector potential is proportional to the acceleration while the 9d vector potential at the bottom of the figure is proportional to the 2nd order derivative of the acceleration.

More general we can write for the magnetic vector potential in 1+9d for a charge ρ making small pertubative movements away from r=0:

Magnetic vector potential in 1+9 dimensions:

$$\vec{A}_9 = \frac{\rho}{\epsilon_o}\mathrm{P}_9 * \vec{v}_t = \frac{\rho}{2\pi^4\epsilon_o}\Bigg(\frac{3!}{0!3!}\,\frac{1}{2^4}\,\frac{1}{r^4}\,\frac{\partial^4\vec{x}}{\partial t^4} + \frac{4!}{1!2!}\,\frac{1}{2^5}\,\frac{1}{r^5}\,\frac{\partial^3\vec{x}}{\partial t^3} + \frac{5!}{2!1!}\,\frac{1}{2^6}\,\frac{1}{r^6}\,\frac{\partial^2\vec{x}}{\partial t^2} + \frac{6!}{3!0!}\,\frac{1}{2^7}\,\frac{1}{r^7}\,\frac{\partial\vec{x}}{\partial t}\Bigg) \tag{31}$$

Where the last (fastest decaying) term corresponds with the classical 3d magnetic vector potential. This last term presents the EM radiation dependent on the acceleration of the charge, while the others terms represent EM radiation dependent on higher order derivatives of the acceleration. (Note that we still have used c=1 here).

VII. CONCLUSION

We found that there are *dimensional dependent* ratios between the EM radiation and Coulomb field, and one should expect, due to the $1/r^n$ nature, to be able to measure imprints of any propagation through higher dimensional structures at arbitrary scale down to Planck's scale. One should be able to do so with table top equipment rather than with LHC scale experiments. We further found that the slowest decaying EM radiation term does not depend, as it does in 3d, on the acceleration of the charge but on higher time derivatives of the acceleration. This provides an extra means to determine the specific dimensionality.

This work has implications for propagators other than the photon as well, like the similar graviton propagators but also for the propagators of non-zero mass particles. It may have implications for the inertial properties of mass itself, in higher dimensional spaces, if we use classical arguments which relate *radiation reaction* with inertial mass. That is, inertial mass would not only depend on acceleration but also on the derivatives of the acceleration.

Work with the simulator is ongoing. One result, with regard to massive propagators, is that we have not found any propagation outside the light cone whatever. Not in any dimensional space. This contradicts with often heard arguments that the propagators "leak" outside the lightcone within a range of m^{-1} . (Note that this range would be infinite in the limit case of massless particles!) Zee (I.23) or Peskin & Schroeder (2.52). The latter use an anti-particle argument to cancel out the propagation outside the light-cone.

We can follow these claims back to Feynman's 1949 landmark paper "The theory of positrons" where he found the Hankel function in the tables as a solution of the propagator in configuration space. This is however a complex combination of the two real functions: $H_1^{(2)}(ms) = J_1(ms) - iY_1(ms)$, the Bessel functions. This is the "Bessel equivalent" of the complex exponential $\exp(-ims) = \cos(ms) - i\sin(ms)$ The Hankel function is complex inside the light-cone for real masses but becomes real for imaginary masses or outside the lightcone.

It is this, exponentially decaying function, which is responsible for the supposed "leaking" outside the light-cone. This leaking however does never occur in our simulations. One can hardly blame somebody, in the pre-computer days of 1949, taken into account that the work in this paper would never have succeeded without the use of extensive numerical simulations.